

DECISION-MAKING, RISK AND THE UTILITY FUNCTION

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Have you ever tried to analyze a problem and after much effort you reach a conclusion that just doesn't seem correct...so you reject it in favor of another choice that "felt better" to you? Does this mean that it is a waste of time to arrive at a decision by analytical means? If this sounds familiar, then it might be beneficial to become acquainted with the concepts developed in Utility Theory.

Our daily lives consist of a continuous stream of decision events. Many of these decisions tend to be of little consequence and have to do more with personal preference or convenience than substance. In fact, typically these decisions are made without even consciously thinking about making a decision. However, occasionally decisions must be made that carry significance that can have far-reaching impact and implications. When faced with such situations in your business or personal life, it would be helpful to have a basis for making those decisions other than intuition or chance. In other words, especially in these uncertain times, is there a systematic way of improving decision-making especially when there is incomplete or conflicting information?

In fact, there are techniques for addressing this problem. Decision Analysis and Management Science has evolved substantially over the last 25 years. Much of this has been driven by the development of the personal computer, but it has also been required by the increasing complexity of our society. While it isn't possible for this article to even survey the theory and practice of the field, it might be helpful to highlight one or two techniques that can be used to sort out decision factors.

When faced with an important decision, there is a mix of at least four factors. First, there is some degree of uncertainty surrounding the question under consideration. Usually, this is related to the fact that there is incomplete information upon which to base the decision. Generally, the objective is to achieve or maximize some type of gain. At the same time, there may also be a need to avoid or minimize an associated loss. Associated with these factors, there exists a degree of risk that the desired gain may not be achieved. In difficult situations not only is the gain not achieved, but also the loss that was feared could occur. With the economic situation that has developed over the past year or two, many people who have money invested in a 401k or other investment fund can relate to this problem description.

One technique frequently used in an attempt to balance these factors is the Utility Function. While a fully developed explanation of utility theory can get somewhat complicated, with a little practice, its routine use can be quite straightforward. Before proceeding with the explanation of the utility function process there are three points that need to be emphasized.

Preliminary considerations

Although it may seem obvious, it is first necessary to carefully define the problem statement. It is not unusual for people to quickly form the question to be solved, but how one defines or frames the problem can profoundly influence the outcome. Thus, make sure that you are trying to answer the right question. Helpful thoughts along this line are to note:

- What triggered the need for a decision?
- What are the known constraints?
- What are the essential elements of the problem?

Closely related to the problem definition is the clarification of objectives that you wish to achieve through the decision. This is important because the objectives form the basis by which the alternatives will be evaluated.

Second, when risk/reward evaluations and tradeoffs are made, they by nature involve a level of subjectivity. One person may be risk-averse while another may be risk-tolerant or risk-seeking. Accordingly, the "right" decision may be dependent on the person asking the question or evaluating the options. This can be an important point to remember if several people are involved in making the final decision.

Finally, it should be noted that a good decision may not equal a good outcome and, conversely, a good outcome may result from a poor decision. The process of good decision-making increases the probability of achieving a good

outcome, but it does not guarantee it. Uncertainty and risk have real effects and can influence the outcome outside of the control of the analyst. Using a structured decision-making process can also highlight the important success factors which can improve asset allocation as well as determine “stopping rules” to minimize loss if an outcome is not achieving the desired results.

Why use Utility Functions?

One of the primary reasons for the development of Utility Theory is that sometimes common alternatives analysis techniques don’t adequately capture the risk associated with a decision, relative to the decision-makers risk tolerance. Common deterministic methods of comparing alternatives such as net present value, internal rate of return, and cost-benefit analysis generally are not set up to consider risk and uncertainty in their analysis. Accordingly, if the decision-maker is aware that risk exists and it has not been adequately factored into the analysis, then his confidence in the validity of the final decision may be weakened.

As an example, suppose that a manager must decide on a process improvement, and two solutions may exist. After doing a net present value (NPV) analysis, Alternative A has an NPV of \$400,000 and Alternative B has a NPV of \$200,000. However, the decision-maker knows that if a major client cancels its order, the NPV of Alternative A is -\$100,000 while Alternative B is \$75,000. In this situation, Alternative A clearly is the best solution so long as the major client remains stable. However, Alternative B has much less risk in that it will not result in a loss regardless of the client action. In other words the appropriate investment decision will be strongly influenced by the amount of risk that the decision-maker deems appropriate and how sensitive the company is to operating at a loss with respect to the investment horizon.

It is for this type of situation that stochastic or probabilistic methods have been developed. While many of these approaches can become mathematically intensive, the application of Utility Theory can be fairly straightforward. An interesting characteristic is that it describes how individuals should make decisions as well as predicting how they do make decisions.

How is a Utility Function developed?

In formal settings, the basic data that will be used for decision-making will most likely be developed by using Expected Monetary Value (EMV), with or without a decision tree. This process can also be used on non-monetary decisions that are common to daily life. However, in order to develop a utility function, the alternatives must be defined and clarified in an orderly fashion. A convenient means of doing this is through the use of a decision tree, which identifies the different options under consideration and assigns a probability of occurrence to each. From this information, a Utility Function is developed that translates each of the possible results into a non-monetary measure known as a “utility.” The “utility” of this result represents the total value or desirability of a decision alternative to the decision-maker. Although not required, a convenient way to do this is to assign a utility value between 0 and 1 to represent the desirability of the alternative, where 0 is least desirable and 1 is most desirable. A simple means of developing a utility function is as follows:

- carefully consider each alternative and the desirability of it coming to pass. Then, assign to it a utility (or desirability) value (between 0 and 1);
- estimate the probability of the alternative coming to pass;
- using the EMV or other ranking result for each alternative, multiply by the utility value and the probability of occurrence to obtain an overall utility; then
- compare alternatives to select the alternative with the highest utility.

The key in this process is to find the “point of indifference” between the competing alternatives and states of nature. At its core, this is an intuitive balancing of the characteristics of the options. This is sometimes difficult, but it forces the decision-maker to quantify his viewpoint. In its simplest form, a rudimentary spreadsheet can be used as follows:

Alternative	Condition	Occurrence Probability	Desirability	Utility
A	1	0.7	0.5	0.35
	2	0.3	0.4	0.12
Overall desirability (sum):				0.47
B	1	0.8	0.7	0.56
	2	0.2	0.2	0.04
Overall desirability (sum):				0.6

In this simple case, two options have two different outcomes with different probabilities of occurrence. The decision-maker has a different sense of the desirability of each outcome. By structuring the problem in this way, one can see that Alternative B has the greatest overall opportunity of a desirable outcome, even though outcome B.2 has the lowest desirability. Looking at the problem in this way, if the

decision-maker “doesn’t like” the result, they can adjust the desirability estimates to be more in line with actual perceptions. Also, by looking at the table one may be able to adjust the occurrence probability by seeing the importance of various components and exerting more effort to improve the possibility of that action coming to pass. Note that this particular example didn’t include a monetary (quantitative) component. Thus, the technique can be used on qualitative decision-making.

This process can be extended to decisions using quantitative characteristics (e.g., money) without a great deal of difficulty. The primary difference is that the EMV of each alternative and outcome is multiplied by the probability of occurrence and utility (desirability) estimate. Because by convention the worst projected outcome is assigned “0” and the best outcome is assigned “1,” the other options must be scaled between them.

To illustrate using the example above, the pertinent values can be estimated as follows:

$$\begin{aligned} \text{Most desirable outcome } U(\$400,000) &= 1 \\ \text{Least desirable outcome } U(-\$100,000) &= 0 \end{aligned}$$

Comparing to Alternative B (Outcome 1 = \$200,000), the decision-maker believes that relative to making \$400,000 or losing \$100,000, he would rate being given the \$200,000 at 0.7 desirable. That is, it’s not as desirable as getting \$400,000, but it’s much more desirable than losing \$100,000. Thus, point of indifference, or utility, is 0.7. Likewise, when considering Alternative B (Outcome 2 = \$75,000) the decision-maker rates that option at 0.4, because it’s not nearly as good as making \$400,000, but it’s still much better than losing \$100,000.

In basic mathematical terms this is expressed as:

$$U(\text{Option}) = U(\text{Best Outcome})p^* + U(\text{Worst Outcome})(1-p^*)$$

where: $U(\text{Base})$ = Utility of base case under consideration

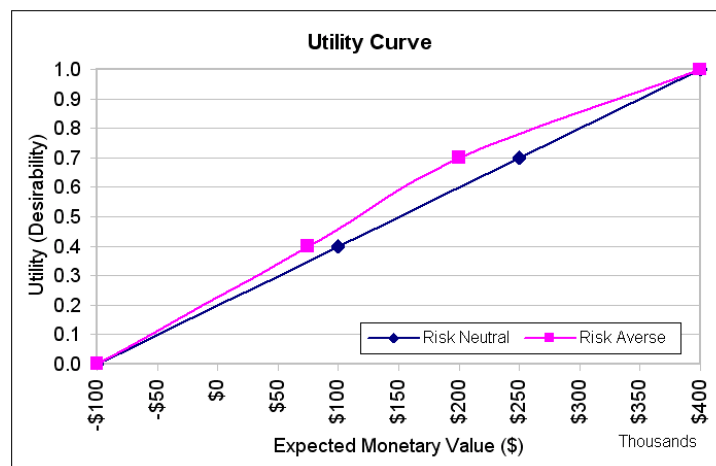
p^* = point of indifference between the alternatives (between 0 and 1)

$$U(\$200,000) = \$400,000 * 0.7 + (-\$100,000) * 0.3 = \$250,000$$

$$U(\$75,000) = \$400,000 * 0.4 + (-\$100,000) * 0.6 = \$100,000$$

Examination of these results indicates that the decision-maker is risk-averse, which can be seen by noting that the he is willing to accept \$200,000 for option B, when the EMV for Option A is \$250,000. Likewise, he is willing to accept \$75,000 for the less desirable case of Option B, rather than Option A which had an EMV of \$100,000. Another way of expressing the result is that the decision-maker was willing to pay a risk premium of \$50,000 or \$25,000 for the two conditions.

This can be shown graphically by plotting the individual points in what is referred to as a Utility Curve. By entering the chart at a particular utility value, one is able to see how the decision-maker values a particular risk for the defined problem. If the curve lies above the straight line (risk-neutral), the decision-maker tends to be risk-averse; if below the straight line the tendency is to be risk-seeking.



While this process can become more involved for more complicated decisions, the real value is that it forces one to examine the potential outcomes in relation to the risk involved. Without a doubt, much of this is subjective, but that is the whole point. As much as we may believe that we are completely objective in our decision-making, the fact of the matter is that there is always a subjective element to it, especially if uncertainty and risk are factors in the decision. As an example, the recent volatility in the stock market will likely have many people reevaluating their risk profile. This is one tool that can help explore their perceptions of risk.

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